B.Sc. Part I (Hons.) 2nd Paper

Integral calculus

Reduction formulae

1. $I_n = \int \tan^n x \, dx$ —— (1)

$\Rightarrow I_n = \int \tan^{n-2} x \cdot \tan^2 x \, dx$

$= \int \tan^{n-2} x \cdot (\sec^2 x - 1) \, dx$

$\Rightarrow I_n = \int \tan^{n-2} x \cdot \sec^2 x \, dx - \int \tan^{n-2} x \, dx$

$\Rightarrow I_n = I_1 - I_{n-2}$ —— (2)

where $I_1 = \int \tan^{n-2} x \cdot \sec^2 x \, dx$

Put $\tan x = z$

$\Rightarrow \sec^2 x \, dx = dz$

and $\tan^{n-2} x = z^{n-2}$

$\Rightarrow I_1 = \int z^{n-2} \, dz$

$= \frac{z^{n-1}}{n-1}$

$\Rightarrow I_1 = \frac{\tan^{n-1} x}{n-1}$

Putting this value in (2)

$$I_n = \frac{\tan^{n-1} x}{n-1} - I_{n-2}$$

$$\Rightarrow I_n + I_{n-2} = \frac{\tan^{n-1} x}{n-1}$$

Ans

Q. If $I_n = \int_0^{\pi/4} \tan^n x \, dx$, Prove that.

$$I_n + I_{n-2} = \frac{1}{n-1}$$

Soln We have already proved that

$$\int \tan^n x \, dx + \int \tan^{n-2} x \, dx = \frac{\tan^{n-1} x}{n-1}$$

Take limit 0 to $\pi/4$,

$$\Rightarrow I_n + I_{n-2} = \frac{1}{n-1} \left[\tan^{n-1} x \right]_0^{\pi/4}$$

$$= \frac{1}{n-1} \left[\tan^{n-1} \frac{\pi}{4} - \tan^{n-1} 0 \right]$$

$$\Rightarrow I_n + I_{n-2} = \frac{1}{n-1} [1 - 0].$$

$$\Rightarrow I_n + I_{n-2} = \frac{1}{n-1}$$ Pd